NAG Toolbox for MATLAB

f08cs

1 Purpose

f08cs computes a QL factorization of a complex m by n matrix A.

2 Syntax

```
[a, tau, info] = f08cs(a, 'm', m, 'n', n)
```

3 Description

f08cs forms the QL factorization of an arbitrary rectangular real m by n matrix.

If $m \geq n$, the factorization is given by:

$$A = Q\begin{pmatrix} 0 \\ L \end{pmatrix},$$

where L is an n by n lower triangular matrix and Q is an m by m unitary matrix. If $m < n$ the factorization is given by

$$A = QL,$$

where L is an m by n lower trapezoidal matrix and Q is again an m by m unitary matrix. In the case where $m > n$ the factorization can be expressed as

$$A = \begin{pmatrix} Q_1 & Q_2 \end{pmatrix}\begin{pmatrix} 0 \\ L \end{pmatrix} = Q_2 L,$$

where Q_1 consists of the first $m - n$ columns of Q, and Q_2 the remaining n columns.

The matrix Q is not formed explicitly but is represented as a product of $\min(m, n)$ elementary reflectors (see Section 3.2.6 in the F08 Chapter Introduction for details). Functions are provided to work with Q in this representation (see Section 8).

Note also that for any $k < n$, the information returned in the last k columns of the array **a** represents a QL factorization of the last k columns of the original matrix A.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D 1999 *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **a**(**lda,**$*$) **– complex array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The m by n matrix A.

5.2 Optional Input Parameters

1: **m – int32 scalar**

Default: The first dimension of the array **a**.

m, the number of rows of the matrix A.

Constraint: $\mathbf{m} \geq 0$.

2: **n – int32 scalar**

Default: The second dimension of the array **a**.

n, the number of columns of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, work, lwork

5.4 Output Parameters

1: **a(lda,$*$) – complex array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

If $m \geq n$, the lower triangle of the subarray $\mathbf{a}(m-n+1:m, 1:n)$ contains the n by n lower triangular matrix L.

If $m \leq n$, the elements on and below the $(n-m)$th superdiagonal contain the m by n lower trapezoidal matrix L. The remaining elements, with the array **tau**, represent the unitary matrix Q as a product of elementary reflectors (see Section 3.2.6 in the F08 Chapter Introduction).

2: **tau($*$) – complex array**

Note: the dimension of the array **tau** must be at least $\max(1, \min(\mathbf{m}, \mathbf{n}))$.

The scalar factors of the elementary reflectors (see Section 8).

3: **info – int32 scalar**

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **m**, 2: **n**, 3: **a**, 4: **lda**, 5: **tau**, 6: **work**, 7: **lwork**, 8: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

The computed factorization is the exact factorization of a nearby matrix $(A + E)$, where

$$\|E\|_2 = O(\epsilon)\|A\|_2,$$

and ϵ is the ***machine precision***.

8 Further Comments

The total number of real floating-point operations is approximately $\frac{8}{3}n^2(3m - n)$ if $m \geq n$ or $\frac{8}{3}m^2(3n - m)$ if $m < n$.

To form the unitary matrix Q f08cs may be followed by a call to f08ct:

```
[a, info] = f08ct(a, tau);
```

but note that the second dimension of the array **a** must be at least **m**, which may be larger than was required by f08cs.

When $m \geq n$, it is often only the first n columns of Q that are required, and they may be formed by the call:

```
[a, info] = f08ct(a, tau, 'k', n);
```

To apply Q to an arbitrary complex rectangular matrix C, f08cs may be followed by a call to f08cu. For example,

```
[c, info] = f08cu('Left','Conjugate Transpose', a, tau, c);
```

forms $C = Q^{\mathrm{H}}C$, where C is m by p.

The real analogue of this function is f08ce.

9 Example

```
a = [complex(0.96, -0.8100000000000001), complex(-0.03, +0.96), complex(-
0.91, +2.06), complex(-0.05, +0.41);
     complex(-0.98, +1.98), complex(-1.2, +0.19), complex(-0.66, +0.42),
...
    complex(-0.8100000000000001, +0.5600000000000001);
     complex(0.62, -0.46), complex(1.01, +0.02), complex(0.63, -0.17),
complex(-1.11, +0.6);
     complex(-0.37, +0.38), complex(0.19, -0.54), complex(-0.98, -0.36),
complex(0.22, -0.2);
      complex(0.83, +0.51), complex(0.2, +0.01), complex(-0.17, -0.46),
complex(1.47, +1.59);
             complex(1.08,  -0.28),  complex(0.2,  -0.12),  complex(-
0.07000000000000001, +1.23), complex(0.26, +0.26)];
[aOut, tau, info] = f08cs(a)
```

```
aOut =
   -0.1733 - 0.3535i    0.0911 + 0.2475i   -0.3728 + 0.4958i   -0.0048 +
0.1360i
   -0.2462 + 0.2361i   -0.4802 - 0.1420i   -0.1765 + 0.1573i   -0.2500 +
0.2066i
   -1.8353               0.4822 - 0.0960i    0.2147 + 0.1362i   -0.3473 +
0.2282i
   -1.0009 + 1.3204i   -1.5755              -0.2486 - 0.2052i    0.0666 -
0.0718i
    0.4157 + 1.3068i   -1.1014 - 0.0597i   -2.9917               0.5272 +
0.4803i
  -0.9548 + 0.7897i  -0.3056 + 0.2217i  -0.0128 + 0.0842i  -2.7650
tau =
   1.5643 - 0.1176i
   1.2513 - 0.1908i
   1.1372 - 0.3465i
```

```
   1.0940 + 0.0940i
info =
           0
```